Pollution and economic growth

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Abstract

We analyse a model of overlapping generations in which clean air, a pure public good, is used as a private input into production. Although production exhibits constant returns to scale, endogenous growth can occur. In a laissez-faire equilibrium, firms generate rents that are the value of the pollution they create. These rents crowd out investment and slow economic growth. Such an equilibrium may not support Pareto optimal allocations, but a Pigouvian tax does. Hence, a pollution tax can yield a double dividend because it reduces pollution and increases growth.

Keywords

Environment, growth, double dividend

1. INTRODUCTION

What limits to economic growth does a concern for the environment entail? The popular press often implies that growth causes environmental degradation. Some authors emphasize that pollution is an inexorable by-product of industrialization and that increasing material affluence will entail a decreasingly attractive world. Views like these were fuelled by the gloomy picture presented in Meadows *et al.*'s (1972) *The Limits to Growth*. Others claim that countries imposing high environmental standards spur research in pollution abatement, develop human capital, and increase growth. Indeed, the World Bank's (1992) report *Development and the Environment* emphasizes that care for the biosphere will require economic growth and that growth cannot take place in a deteriorating environment.

The purpose of this paper is to show that appropriate environmental policies might actually spur growth. Economists often think of a clean environment as a luxury. Thus, as a country becomes richer, its citizens demand that greater attention be paid to the habitat, and they are willing to impose environmental taxes and regulations as public policy. The model presented below shows that these taxes and regulations might actually pay a 'double dividend'. This bonus arises through a mechanism that has been overlooked: an environmental tax dissipates the rents associated with pollution and mitigates the extent to which they crowd out investment.[1]

The model we use builds on the work of Fisher (1992), who discusses the general properties of an overlapping generations model with constant returns to scale. He shows that the assumption of finite lives makes it necessary for the current price of an investment good to decline as the economy grows. Hence, if agents do not live forever, growth cannot occur in a one-sector economy, and a two-sector model describes the asymptotic behaviour of a wide class of growing economies.

We utilize the model of overlapping generations since it lends itself naturally to the analysis of the effects of governments' environmental policies when agents' actions have consequences that endure beyond their own lifetimes. Several authors have analysed similar issues. Mourmouras (1993) studies a model of renewable resources and shows that equilibrium may entail perpetually declining standards of living. Using a model with finitely many generations, Howarth (1991) explores the trade-off between economic efficiency and social equity in a model with exhaustible resources. Howarth and Norgaard (1992) argue that sustainable economic development is essentially a matter of inter-generational equity. We show that this issue, in the guise of the distribution of pollution rights, has important effects for economic growth and thus influences the well-being of those not yet born.

We demonstrate that Pigouvian pollution taxes with compensating lump-sum transfers can represent a Pareto improvement over the laissez-faire equilibrium. Batina (1990) argues that it is not realistic to assume that a government has this kind of control in a dynamic economy. Still, we describe a simple rule that can be implemented in part by using publicly available information: the government should compensate the owners of firms for the capital losses arising from pollution taxes. There is no simple rule for these taxes because the public's willingness to pay for pollution abatement may depend, in general, upon interest rates and the profile of aggregate income.

The second section describes the model and its laissez-faire equilibrium. The third analyses properties of this laissez-faire equilibrium and its inefficiencies. These distortions arise because environmental externalities have not been internalized. The fourth section analyses a dynamic profile of Pigouvian taxes and shows that such taxes, coupled with a sequence of lump-sum transfers, can entail a Pareto improvement over the laissez-faire equilibrium. The fifth section presents a simple analytical example, while the sixth presents our conclusions.

2. THE MODEL AND ITS LAISSEZ-FAIRE EQUILIBRIUM

There is an infinite sequence of generations with one person born in each period. The initial 'old' agent lives for only one period, but every other person lives for two periods. We shall use the convention that a subscript refers to a dated commodity and a superscript to an agent. We assume that the population and labour force are constant, although our results generalize easily when population growth is exogenous.

The initial generation is endowed with the economy's stock of capital, and every other person is endowed with L units of labour in the first period of his life and nothing else. This description of endowments shows that the path of the economy describes per capita economic growth. Also, a worker must save part of his wage to finance consumption in old age.

We model pollution as a pure public consumption good that is a private input into production. Just as labour is leisure used as an input, so is pollution-clean air used in production. The amount of clean air in the economy is in limited supply in every period. A firm's use of clean air is reflected in its production technique, and one firm's inputs cannot be used by another. Aggregate environmental degradation is determined by the production decisions of all firms in the economy.

Firms' production decisions give rise to a pollution profile described by

$$S = (S_1, S_2, \ldots)$$

a uniformly bounded sequence. This description of the environment is quite general. This specification can entail a declining, constant, or increasing but bounded flow of clean air per period. The long-run characteristics of the environment and the production processes will be described by the limiting behaviour of this sequence. Although we have not given an explicit law of motion for clean air, the reader can think of this sequence as a reduced form for a more complicated dynamic specification.

Let c_t^h be agent h's consumption at time t. Then preferences are represented by the utility functions: $u^0(c_1^0, -S_1)$ for the original generation and $u^t(c_t^t, c_{t+1}^t, -S_t, -S_{t+1})$ for the generation $t > 0$. These functions are increasing in each of their arguments, showing that agents care about their own consumption and the disutility they suffer from pollution during the course of their lives. Note that agents in generations $t-1$ and t suffer disutility from S_t, the aggregate amount of clean air used by all firms in period t. Thus, clean air is a pure public good, and aggregate pollution is global, not local.

There are two sectors in the economy, each sector with a continuum of firms. Firms in sector 1 produce final goods and services, and those in sector 2 produce intermediate investment goods. The production function for firm $j \in [0,1]$ in sector $i \in \{1,2\}$ is

$$Q_{t,i}(j) = F_i(K_{t,i}(j), L_{t,i}(j), S_{t,i}(j))$$

where $K_{t,i}(j)$ is the firm's input of capital, $L_{t,i}(j)$ is that of labour, and $S_{t,i}(j)$ is that of clean air used as an input, or pollution emitted, all at time t. We will assume that each production function is homogeneous of degree one in its inputs and is weakly increasing in each of its arguments; thus there are no increasing returns to scale, nor is there exogenous technical progress. Economic growth occurs because of capital accumulation, but one need not think of the second sector simply as producer durable goods. Instead, one might consider capital as an aggregate reproducible factor whose private and social rates of return do not differ. We assume constant returns to scale in each sector in order to isolate the effects of pollution as a negative externality in the growth process. Moreover, recent empirical studies do not support the hypothesis of increasing returns; see Benhabib and Jovanovic (1991) or Mankiw *et al.* (1992).

Each type of firm in each sector creates a fixed amount of pollution in each period. Let $\mu_i(j)$ be a stationary measure describing the distribution of firms in sector $i \in \{1,2\}$. Then the aggregate flow of pollution created in each period is

$$S_t = \int S_{t,1}(j)\, d\mu_1 + \int S_{t,2}(j)\, d\mu_2$$

where each integral is taken over the relevant support and $d\mu_i$ denotes the obvious density. Each firm creates an infinitesimal amount of pollution, but firms create a negative externality in the aggregate. A firm's pollution is exactly analogous to the role of land in Ricardo's theory of rents. In particular, firms' choices of capital and labour are determined on the *intensive margin*, whereas which particular firms operate is determined on the *extensive margin*. Also, since the production functions are linearly homogeneous, if clean air is a necessary input, then a firm experiences decreasing returns to scale in capital and labour, the marketed factors of production.

It is important to describe the pattern of ownership of the firms because they generate rents. We will assume that the initial generation owns the economy's entire stock of equity in all firms in period 1. Hence, agent $t \geq 1$ will save some of his wage, invest in new capital, and purchase equity. This process repeats itself in each period, as the older generation divests itself of equity and the young use savings to purchase the investment good and equity in both kinds of firms. In the absence of a pollution tax, the 'dividend' from having purchased a firm in period t is the value of its output that is attributed to the use of clean air in period $t + 1$.

The law of motion for capital is $K_{t+1} = \int Q_{t,2}(j)\, d\mu_2$, where we are implicitly assuming that capital depreciates completely in each period; this assumption underscores the notion that a period corresponds to the economic life of a worker. Of course, the economy satisfies the resource constraints

$$\int K_{t,1}(j)\, d\mu_1 + \int K_{t,2}(j)\, d\mu_2 \leq K_t \quad \text{and} \quad \int L_{t,1}(j)\, d\mu_1 + \int L_{t,2}(j)\, d\mu_2 \leq L$$

We are now in a position to describe the decisions of firms and consumers. Let $P_{t,i}$ be the present price of a claim against the good $i \in \{1,2\}$, W_t be the analogous price of labour, and R_t be the analogous rentals rate. We use the normalization that $P_{1,1} = 1$; hence, $P_{t,i}/P_{t+1,i}$ is a commodity's own rate of interest, and $P_{t,2}/P_{t,1}$ is the current price of an investment good.

A firm's accounting profits at time t are $\Pi_{t,i}(j) = P_{t,i}Q_{t,i}(j) - R_tK_{t,i}(j) - W_tL_{t,i}(j)$. In each period, a firm takes prices as given and chooses inputs to maximize profits. Of course, since each production function is linearly homogeneous, a firm's accounting profits are the rents that accrue to its use of clean air. Since there is a continuum of firms, shutting down any one firm will entail a complete capital loss for its owners but will have no effect on aggregate pollution.

The value of a firm in sector i at the end of time t is $V_{t,i}(j) = \sum_{s=t}^{\infty} \Pi_{s,i}(j)$, where we have imposed a 'no-bubbles' condition on the asset markets. Thus a firm's equity is equal to the present value of the rents it generates from the pollution it creates. Shares are sold *ex dividend*, and the present value of a firm's equity satisfies $V_{t,i}(j) = V_{t+1,i}(j) + \Pi_{t,i}(j)$, the familiar condition that a share of stock is worth the present value of the sum of dividends and expected capital gains. Thus, profit maximization in each period is equivalent to choosing a production plan that maximizes the value of equity at each point in time, taking the use of clean air as given.

Let $V_{t,i} = \int V_{t,i}(j)\, d\mu_i$ be the present value of the equity in industry $i \in \{1,2\}$. Then the present value of agent h's income is $Y^0 = R_1K_1 + \sum_{i=1}^{2} V_{1,i}$ and $Y^t = W_tL$ for $t > 0$. Agent 0 has income accruing from the ownership of the initial capital stock and the economy's stock of equity. Any other agent commands the discounted value of labour income. Taking prices as given, each chooses a consumption plan and asset holdings subject to his present-value wealth constraint. He uses claims on capital and shares of equity as stores of value to finance consumption in the final period of his life.

Let $Q_{t,i} = \int Q_{t,i}(j)\, d\mu_i$. Then, taking each firm's pollution profile as given, a perfect foresight laissez-faire equilibrium is a list of prices and corresponding aggregate quantities

$$\{(P_{t,1}, P_{t,2}, W_t, R_t)\}_{t=1}^{\infty} \quad \text{and} \quad \{(Q_{t,1}, Q_{t,2})\}_{t=1}^{\infty}$$

satisfying six conditions. First, each consumer chooses asset holdings and a consumption profile to maximize utility, taking as exogenous the stream of pollution he faces during his life. Second, firms maximize profits period by period, again taking their use of clean air as given. Third, the economy's resource constraints are satisfied. Fourth, materials balances hold. Fifth, asset markets clear. Sixth, the initial stock of capital and ownership of equity is given.

The definition of a laissez-faire equilibrium takes as given an arbitrary pattern of pollution among all firms. This pattern matters because it does not assume that clean air is rationed efficiently. We will show below that the asymptotic share of pollution in the investment sector is zero in a growing economy. Hence, the long-run behaviour of such an economy is influenced largely by the use of clean air in the consumption sector. We now turn our attention to how the growth rate of the economy is affected when claims to clean air have been allocated implicitly through the ownership of firm equity.

3. PROPERTIES OF THE LAISSEZ-FAIRE EQUILIBRIUM

First, we state a result describing the asymptotic share of pollution in the investment sector.

Proposition 1: If the supply of clean air is bounded, then the economy exhibits sustained growth only if the asymptotic share of pollution in the investment sector is zero.

Proof: See the Appendix for the proof.

Rebelo (1991) and Fisher (1992) have argued that it is appropriate to think of the investment sector as the engine of growth for this economy. If the amount of pollution that the economy can sustain is bounded, then it is necessary for that engine to be green. The same conclusion, however, may not be true in the consumption sector, where the asymptotic share of pollution may be greater than zero.

Since capital must be strongly productive[2] for this economy to grow, we will assume henceforth that $\lim_{K_{t,2} \to \infty} \partial F_2 (K_{t,2}, L_{t,2}, S_{t,2})/\partial K_{t,2} = \Gamma > 1$. The asymptotic marginal product of capital is independent of the labour force and the supply of clean air because both are bounded by assumption. In equilibrium $P_{t,2}/P_{t+1,2}$ is the marginal rate of transformation of current into future capital. This assumption about the second sector implies that $\lim_{t \to \infty} P_{t,2}/P_{t+1,2} = \Gamma$ in a growing economy. Also, in such an economy $\lim_{t \to \infty} P_{t,2}/P_{t,1} = 0$. Our assumption about production in the investment sector is necessary for growth, but it is not sufficient. We need to describe the agents' savings decisions more fully.

To that end, let us define $1 + i_{t+1} \equiv P_{t,1}/P_{t+1,1}$, the only relative price that matters for savings. Then the consumer's decisions are summarized by a function $\sigma^t (Y^t, i_{t+1}, -S_t, -S_{t+1})$, where savings depend upon income, the real interest rate, and the aggregate profile of environmental degradation facing $t > 0$. Equilibrium in the market for the investment good is expressed by

$$\sigma^t (W_t L, i_{t+1}, -S_t, -S_{t+1}) = P_{t,2} K_{t+1} + \sum_{i=1}^{2} V_{t+1,i} \tag{1}$$

which states that agents must use claims on capital and firms' shares as stores of value. Equation (1) states that agent $t > 0$ acquires full ownership of both firms at the end of the first period of his life.

Now consider a balanced growth path. Since heterogeneous savings decisions will surely complicate the analysis, we will impose that these generations have identical preferences and drop the superscript modifying the savings function. Then (1) implies that such a balanced path is characterized by an increasing aggregate capital stock with constant physical shares allocated to each sector. The gross rate of growth of the physical capital stock is $G \equiv K_{t+1}/K_t$.

Since the production functions for the two sectors are different, they may have disparate physical rates of growth. But, on a balanced path, the relative price of the investment good offsets this differential. The consumption sector grows at the rate $\lim_{t\to\infty} Q_{t+1,1}/Q_{t,1} \equiv G_1$, which depends upon the rate of accumulation of capital and the profile of pollution abatement. Since $\lim_{t\to\infty} P_{t+1,2}\, K_{t+1}/P_{t,2}\, K_t = G/\Gamma$ and the marginal propensity to save is constant, equation (1) describes balanced growth only if $\lim_{t\to\infty} P_{t,1}/P_{t+1,1} = \Gamma G_1/G$. Hence, $1 + i = \Gamma G_1/G$ is the real interest rate, and the value shares of consumption and investment in national income are constant.

Also, we will assume that the asymptotic shares of capital, labour and clean air in the first sector are all well defined. Let these shares be $\theta_{1,K}$, $\theta_{1,L}$ and $\theta_{1,S}$ respectively. They may depend upon the pollution per worker generated by that sector, but postulating that they are well defined is largely equivalent to assuming that the pollution generated by the consumption sector displays no cycles and that the production function in that sector is otherwise well behaved.

Finally, since we are interested in studying balanced growth paths, it seems innocuous enough to assume the marginal propensity to save from permanent income converges to a constant. We will allow this rate to depend upon the entire profile of environmental degradation that the economy faces. Recalling that the long-run real interest rate is a constant and that each agent's permanent income is the present value of his wage bill, we can define: $\lim_{t\to\infty} \sigma(W_tL, i_{t+1}, -S_t, -S_{t+1})/W_tL \equiv \sigma_Y(i, -S)$, the asymptotic marginal propensity to save. Using (1) and noting that a balanced path is characterized by constant physical shares of capital in both sectors, we can solve for the growth rate of the capital stock

$$G = \Gamma \frac{\theta_{1,L}\sigma_Y(i, -S)}{\theta_{1,K} + \theta_{1,L}\sigma_Y(i, -S) + \theta_{1,S}} \tag{2}$$

Now we state the following.

Proposition 2: Assume that the long-run marginal propensity to save is not decreasing in the interest rate. Then a unique laissez-faire equilibrium exists, and the growth rate of capital is described by (2).

Proof: See the Appendix for the proof.

Along a balanced growth path the physical share of capital devoted to the first sector is constant; then logarithmic differentiation of $F_1(K_{t,1}, L_{t,1}, S_{t,1})$

shows that its growth depends positively upon the economy's rate of capital accumulation and negatively upon the rate of pollution abatement in the first sector. Thus, the utility of a representative generation can become arbitrarily large if both $G > 1$ and the effects of environmental degradation on utility are not too deleterious. For any growth rates of the capital stock and the consumption sector, the real interest rate adjusts so that (2) holds.

We conclude this section with the observation that a laissez-faire equilibrium may not support Pareto optimal allocations. Consider the static efficiency criterion

$$-\sum_{h=t-1}^{t} (\partial u^h/\partial c_t^h)^{-1}\, \partial u^h/\partial S_t = \partial F_1/\partial S_{t,1}$$

Since the amount of clean air used by each firm is arbitrary, this equation may not hold. As long as the total marginal disutility of pollution is great enough, people will be willing to give up consumption until the shadow value of clean air in production is sufficiently high. We have just shown the following.

Proposition 3: In a laissez-faire equilibrium, the economy may not support Pareto optimal allocations.

4. POLLUTION TAXES AND ECONOMIC GROWTH

A market for pollution rights is missing in this economy. Since fixed amounts of pollution are created by each firm, such rights fall by default to their owners. Thus, firms' shares have positive value, and (1) shows that they crowd out investment. A conventional market for pollution rights cannot exist because clean air is a pure public good. Thus, there is a compelling case for a Pigouvian pollution tax.

Consider a pollution fee whose present price is Λ_t. Then

$$\Lambda_t/P_{t,1} = -\sum_{h=t-1}^{t} (\partial u^h/\partial c_t^h)^{-1}\, \partial u^h/\partial S_t = \partial F_1/\partial S_{t,1} \tag{3}$$

describes the static optimality criterion for this shadow value. Equation (3) states that a Pigouvian tax equates the marginal willingness to pay for pollution abatement and the marginal product of clean air.[3]

If pollution abatement is a luxury, then (3) states that the current value of clean air rises. If agents' utility functions are separable in clean air and consumption, then (3) describes a simple rule for the imposition of the Pigouvian pollution tax. The pollution tax is high when the marginal disutility of pollution is high. If pollution is necessary in the production of the consumption good, then the marginal product of clean air and the pollution tax will become arbitrarily large.

Consider a government imposing pollution taxes $\{\Lambda_t\}_{t=1}^{\infty}$. Now firm

$j \in [0,1]$ in sector $i \in \{1,2\}$ has rents $\Pi_{t,i}(j) = P_{t,i}Q_{t,i}(j) - R_t K_{t,i}(j) - W_t L_{t,i}(j) - \Lambda_t S_{t,i}(j)$. Since the input of clean air is priced explicitly, it is chosen endogenously by the firm. Since each sector exhibits constant returns to scale, any firm now earns zero profits in equilibrium. Hence, the aggregate value of firm equity is zero. Also, the choice of inputs that maximize profits is independent of the type of firm, and there is an immediate increase in the output of both sectors as firms switch to production techniques where all inputs are chosen on the intensive margin. Thus, introducing pollution taxes increases the level of gross national product because it rations clean air efficiently.

Again, a *perfect foresight equilibrium* is a list of prices and taxes and aggregate quantities

$$\{(P_{t,1}, P_{t,2}, W_t, R_t, \Lambda_t)\}_{t=1}^{\infty} \quad \text{and} \quad \{(Q_{t,1}, Q_{t,2})\}_{t=1}^{\infty}$$

satisfying six conditions analogous to those describing the laissez-faire equilibrium. The only difference is that firms do not take their use of clean air as given exogenously; instead the use of clean air is determined endogenously by the firms' profit maximization decisions.

Since firms now earn no profits, $\sum_{i=1}^{2} V_{t+1,i} = 0$ in each period. Thus, the analogue of (1) is now $\sigma^t(W_t L, i_{t+1}, -S_t, -S_{t+1}) = P_{t,2}K_{t+1}$, and a given profile of real income and environmental degradation allows more of the reproducible factor to be accumulated in each period. Also, the logic leading to (2) shows that the Pigouvian tax increases the economy's rate of growth most dramatically if the asymptotic share of pollution is large. The Pigouvian taxes hurt the owners of firms, but may benefit both current and future consumers. They benefit future consumers in two ways. First, they increase the rate of accumulation of capital and thus the future real wage. Second, they induce an efficient amount of pollution in each period. Thus, pollution taxes can indeed yield a double dividend.

In the preceding paragraph, we assumed implicitly that these taxes simply created a surplus for the government budget. Thus, the Pigouvian taxes essentially usurp the value of the equity owned by the original generation. Of course, the imposition of these taxes is not necessarily a Pareto improvement since it may decrease the utility of this generation. However, there may be a self-financing sequence of lump-sum taxes that enables the government to implement a Pareto improvement.

We will show how to derive such a sequence. Consider an arbitrary distribution of clean air and a corresponding laissez-faire equilibrium $\{(P_{t,1}, P_{t,2}, W_t, R_t)\}_{t=1}^{\infty}$ and $\{(Q_{t,1}, Q_{t,2})\}_{t=1}^{\infty}$. Let $\alpha^0 = u^0(c_1^0, -S_1)$ and $\alpha^t = u^t(c_t^t, c_{t+1}^t, -S_t, -S_{t+1})$ if $t > 0$, where these utility functions are evaluated at the equilibrium allocations and the given profile of environmental degradation. Since labour is supplied inelastically, the *expenditure function* for agent $t > 0$ is[4]

$$e^t(P_{t,1}, P_{t+1,1}, -S_t, -S_{t+1}, \alpha^t) \equiv$$
$$\min_{(c_t^t, c_{t+1}^t)} \{P_{t,1}c_t^t + P_{t+1,1}c_{t+1}^t \mid u^t(c_t^t, c_{t+1}^t, -S_t, -S_{t+1}) \geq \alpha^t\}$$

This is the minimal amount of money that agent $t > 0$ needs to achieve utility α^t.

Now let $\{\alpha^h\}_{h=0}^{\infty}$ describe the profile of utilities in a benchmark laissez-faire equilibrium. Recall that Y^h is the present value of an agent's income. Then the sequence $\{\tau^t\}_{t=0}^{\infty}$ is defined by[5]

$$\tau^t = e^t(P_{t,1}, P_{t+1,1}, -S_t, -S_{t+1}, \alpha^t) - Y^t \tag{4}$$

We are looking for a sequence of compensating lump-sum transfers such that no agent would object to leaving the laissez-fair equilibrium with the appropriate compensation. These lump-sum taxes and transfers do not correspond to balanced budgets in every generation. Instead, they entail running generational deficits to compensate those hurt by the Pigouvian taxes and using the surpluses generated by the pollution taxes to service the interest burden on the national debt. This deficit crowds out growth, but it is less than the value of firms' shares if their owners are willing to pay for pollution abatement.

The informational requirements needed to implement this policy are formidable, but a good rule of thumb for the government is: (1) impose a big tax on pollution when society's willingness to pay for pollution abatement is high; and (2) compensate the owners of firms for any capital losses that these pollution taxes entail. This technique is already in use! It is now commonplace in the United States to charge households marginal cost for solid waste disposal and also to compensate land-owners for capital losses if a new landfill is located near the site of their property. These policies can yield a double dividend: they help abate pollution and increase economic growth.

5. A SIMPLE EXAMPLE

This section gives a simple parametric specification of the model. We assume the utility functions $u^0(c_1^0, S) = \log c_1^0 - \psi S_1$ and $u^t(c_t^t, c_{t+1}^t, S) = (1 - \sigma)(\log c_t^t - \xi S_t) + \sigma(\log c_{t+1}^t - \psi S_{t+1})$ for agent $t \geq 1$. The parameter $0 < \sigma < 1$ denotes the marginal propensity to save from permanent income. The important aspect of this utility function is that the marginal willingness to pay for pollution abatement can be determined in a simple way.[6] Also, saving per worker satisfies $\sigma(W_t, i_{t+1}, -S_t, -S_{t+1}) = \sigma W_t$. We impose Cobb–Douglas production functions, with capital's share being unity in the investment goods sector. These assumptions about preferences, production, and pollution are such that the economy is on a balanced path from its first period, and its asymptotic properties describe the actual equilibrium.

Using the utility function and the production function for the first sector, we see that the rule for the pricing of the pollution tax is $\Lambda_t/P_{t,1} = \psi c_t^{t-1} + \xi c_t^t = \theta_{1,S}Q_{t,1}/S_t$, where all the economy's pollution occurs in the first sector. On a balanced path, the terms in this equation grow at the rate $G^{\theta_{1,K}}$; thus the profile of pollution is constant. The pollution tax rises at the same rate as the real wage.

Of course, the pollution tax now implies that $V_{t+1} = 0$ for all $t+1$. The algebra leading to (2) shows that the growth rate now is $G = \Gamma \dfrac{\sigma\theta_{1,L}}{\theta_{1,K} + \sigma\theta_{1,L}}$, an increase from the rate in a laissez-faire equilibrium. In deriving this growth rate, we used the fact that the government has a surplus of $\theta_{1,S}Q_{t,1}$, the entire revenue of the pollution tax in each period. Since $c_t^{t-1}/Q_{t,1} = \theta_{1,K} + \sigma\theta_{1,L}$ and $c_t^t/Q_{t,1} = (1-\sigma)\theta_{1,L}$ on a balanced path, we may conclude the constant amount of pollution is $\hat{S} = \theta_{1,S}/[\psi(\theta_{1,K} + \sigma\theta_{1,L}) + \xi(1-\sigma)\theta_{1,L}]$. Thus, steady-state pollution is decreasing in society's marginal willingness to pay for its abatement.

Does the new growth trajectory represent a Pareto improvement? In a laissez-faire equilibrium, output of the consumption sector is $Q_{1,1} = [(\theta_{1,K} + \theta_{1,S})/(\theta_{1,K} + \theta_{1,S} + \sigma\theta_{1,L})]^{\theta_{1,K}} K_1^{\theta_{1,K}} S_1^{\theta_{1,S}} L^{\theta_{1,L}}$, where we are assuming an identical distribution of pollution among the firms in the first sector. In the equilibrium with the Pigouvian tax and a government surplus, the analogous output is $\hat{Q}_{1,1} = [\theta_{1,K}/(\theta_{1,K} + \sigma\theta_{1,L})]^{\theta_{1,K}} K_1^{\theta_{1,K}} \hat{S}^{\theta_{1,S}} L^{\theta_{1,L}}$, where again $\hat{S}$ is the optimal constant level of pollution. Since the laissez-faire level of pollution was arbitrary, this difference can represent an increase or decrease in the output of the first sector. If the level of pollution is unchanged, output must decrease, since fewer resources are devoted to current consumption and more to investment. Of course, if the original equilibrium had an inefficient distribution of pollution among firms, output may rise once the Pigouvian tax is in place, even if the aggregate pollution is curtailed.

Using the utility of the representative agent in the original generation, the production function in the first sector, and the fact that $c_t^{t-1} = (\theta_{1,K} + \theta_{1,S} + \sigma\theta_{1,L})Q_{t,1}$ in a laissez-faire equilibrium, we can infer that $\mathrm{d}u^0/\mathrm{d}S_1 = \theta_{1,S}/S_1 - \psi$. Thus, if the laissez-faire equilibrium had, relatively, a lot of pollution, the original generation's utility is increasing in pollution abatement. Hence, a sufficient condition for improving the welfare of the original generation is that its marginal willingness to pay for pollution abatement in the laissez-faire equilibrium must be high. Finally, we note that this condition also suffices for the welfare of every generation to rise, since $c_t^{t-1} = G^{\theta_{1,K}(t-1)} c_1^0$. Of course, the parameter ψ must be sufficiently large so that an initial (finite) string of generations prefer the equilibrium with lower pollution and higher investment.

What about an equilibrium with national debt? The pricing of the pollution tax is exactly as in the example with the government surplus, and again the pollution tax implies that $V_{t+1} = 0$ for all $t + 1$. Consider first the case where all firms are identical and are producing the optimal level of pollution $\bar{S}$. Then the laissez-faire equilibrium was already Pareto optimal, and moving to the Pigouvian equilibrium entails no increase in the static efficiency of the economy. In this case, we set $\tau^0 = V_1$ for the initial generation and $\tau^t = 0$ for every other agent $t > 0$. Thus, the equilibrium with national debt replicates the original (efficient) laissez-faire equilibrium. Firms in the economy use their erstwhile dividends to pay the new pollution tax, the government uses these revenues to amortize the national debt, and government bonds serve the same role as a store of value as did firm equity in the original equilibrium.

Now consider an economy in which half the consumption firms create no pollution and the other half created $2\bar{S}$. Of course, the firms that do not pollute would not be profitable, and their decisions on the extensive margin would keep them out of business. In the equilibrium with a pollution tax, new firms enter the first sector and output expands there. Since the aggregate amount of pollution is unchanged, it satisfies the optimality criterion, but there is an immediate increase in output and no resources are drawn away from the investment sector. This gain allows the government to set $\tau^0 < V_1$, and again it can put $\tau^t = 0$ for every other agent $t > 0$. Hence, there is less national debt and an incipient increase in the growth rate, benefiting all current and future generations. Again firms in the economy use their former dividends to pay the pollution tax, and claims against the national debt have a competitive return. But, on a balanced growth path, the present value of the pollution taxes is $\theta_{S,1}\bar{Q}_{1,1}\Gamma/(\Gamma - G)$, which is greater than V_1 and is also increasing in the growth rate. Since the pollution tax creates an incipient rise in growth, the government can amortize the national debt in finitely many periods! Then, the economy will be on a balanced growth path described by the case where it runs perpetual surpluses. Hence, the same sufficient conditions for a Pareto improvement hold in this case.

We have demonstrated that it is sufficient that the owners of firms have a high marginal willingness to pay for pollution abatement for the equilibrium with a Pigouvian tax to effect a Pareto improvement. Also, even if the aggregate amount of pollution is not excessive, the pollution tax will cause a Pareto improvement if there is an initially inefficient distribution of production. Hence we conclude that a pollution tax can increase growth, finance part of the national debt, and improve the welfare of every generation in the economy.

6. CONCLUSIONS

We have described a model with endogenous growth where the environment is a pure public good that is used as a private input into production. In a laissez-faire equilibrium, the owners of firms reap rents that are the entire present value of the stream of pollution that their firms generate. Even though there is no technological progress, this economy can exhibit sustained growth if the marginal propensity to save is high and if these rents do not represent a large share of national income. It is also necessary that the asymptotic share of pollution in the investment goods sector be zero. This sector represents, in a broad sense, a reproducible factor of production whose private and social rates of return are equal. One need not think of the investment sector as heavy industry.

Any pollution tax increases the rate of economic growth and the current level of output because it dissipates the rents that crowd out the accumulation of capital and rations the use of clean air efficiently. A pollution tax will lower the welfare of the owners of firms, so that there may be a need to compensate them. Deciding on the correct profile of Pigouvian taxes is quite a daunting task because it entails judging the marginal benefit of pollution abatement for each agent of infinitely many generations. If a suitable policy can be implemented, it entails an intuitive profile of national debt: the government runs a deficit to compensate the current owners of firms and amortizes the national debt from the future receipts of pollution taxes. Hence, an economy can grow and still be green, and its government can compensate those who suffer from pollution taxes. Such a policy need not increase the present value of the national debt if it is implemented appropriately.

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APPENDIX

Proof of Proposition 1: Since the supplies of both clean air and labour are bounded, the economy displays sustained growth only if there is an $\varepsilon > 0$ such that

$$1 + \varepsilon < K_{t+1}/K_t = \int F_2(K_{t,2}(j), L_{t,2}(j), S_{t,2}(j))\, \mathrm{d}\mu_2/K_t$$

Letting $K_{t,2} = \int K_{t,2}(j)\, \mathrm{d}\mu_2$ and using the fact that $K_{t,2} \leq K_t$, we may infer

$$1 + \varepsilon < \int K_{t,2}(j)[F_2(K_{t,2}(j), L_{t,2}(j), S_{t,2}(j))/K_{t,2}(j)]\, \mathrm{d}\mu_2/K_{t,2}$$

where now we are integrating over the set of firms that use strictly positive amounts of capital. The term in square brackets is the average product of capital, and it is decreasing in the stock of capital. Using L'Hôpital's rule and letting $\lim_{k_{t,2}\to\infty} \partial F_2(K_{t,2}, L_{t,2}, S_{t,2})/\partial K_{t,2} = \Gamma$, we have

$$1 + \varepsilon < \Gamma \int K_{t,2}(j)\, d\mu_2 / K_{t,2} = \Gamma$$

The result then follows from the linear homogeneity of the production function. This completes the proof.

Proof of Proposition 2: Let the shares in the first sector be $\theta_{1,K}$, $\theta_{1,L}$ and $\theta_{1,S}$, as in the text. We can now calculate the limiting value of equity on a balanced path. Since $G < \Gamma$, thus $\sum_{t=1}^{\infty} P_{t,2}K_{t+1}$ is bounded. But then $\{V_{t+1,2}\}_{t=1}^{\infty}$ converges to zero at a faster rate than the other terms in (1) because the asymptotic share of pollution in the investment goods sector is zero. Also, since $1 + i = G_1\Gamma/G$, $\sum_{t=1}^{\infty} P_{t,1}Q_{t,1}$ is bounded too. Then the recursive formula for the present price of equity implies that $\lim_{t\to\infty} V_{t+1,1}/P_{t,1}Q_{t,1} = \theta_{S,1}G/(\Gamma - G)$.

We can now calculate the growth rate implicitly. Rewrite the left-hand side of (1) as $W_tL\sigma(W_tL, i_{t+1}, -S_t, -S_{t+1})/W_tL$. Since the labour force is fixed, the savings function satisfies

$$\lim_{t\to\infty} \sigma(W_tL, i_{t+1}, -S_t, -S_{t+1})/W_tL = \sigma_Y(i, -S)$$

where we have used L'Hôpital's rule. Also, we use the fact that the limiting present value of the wage bill is zero and we are allowing this function to depend upon the entire profile of pollution. Then (1) implies that

$$\theta_{1,L}\sigma_Y(i, -S) = (\theta_{1,K}G + \theta_{1,S}G)/(\Gamma - G) \tag{A1}$$

which describes the growth rate of the capital stock implicitly. The right-hand side of (A1) is a monotonically increasing function of G. Its greatest lower bound is 0, and it has no upper bound as G tends towards the maximal feasible growth rate Γ. Also, as long as higher interest rates do not decrease savings for any S, the left-hand side of (A1) does not increase in G, since $1 + i = G_1\Gamma/G$ and the physical shares of capital in each sector are constant on a balanced path. This completes the proof.

NOTES

1 This result stands in contrast to the work of Bovenberg and de Mooij (1994), who argue that pollution taxes typically exacerbate static economic distortions, and to that of van der Ploeg and Withagen (1991), who find that concern for the environment lowers output in a traditional neoclassical growth model.

2 This is the terminology of Gale and Sutherland (1968). It entails that the investment sector is both linear homogeneous and displays a sufficiently high marginal efficiency of investment so that the economy's capital stock may become arbitrarily large.

3 This tax is quite similar to that described in Diamond (1973).

4 The function for agent 0 is analogous.

5 Of course, the expression for agent 0 is $\tau^0 = e^0(P_{1,1}, S_1, \alpha^0) - Y^0$.

6 Also, since this function is separable, the profile of pollution taxes avoids the dynamic myopia that Strotz (1956) describes for more general preferences.

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